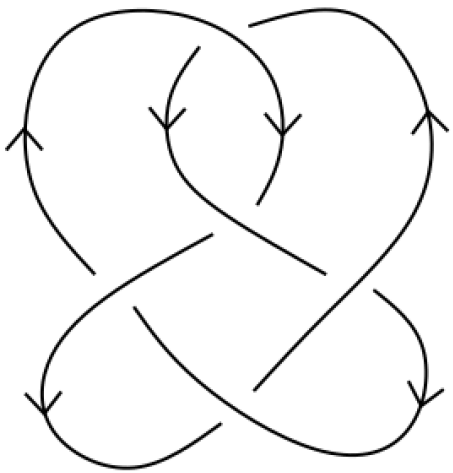
The Minimal Number of Seifert Circles Equals the Braid Index of a Link

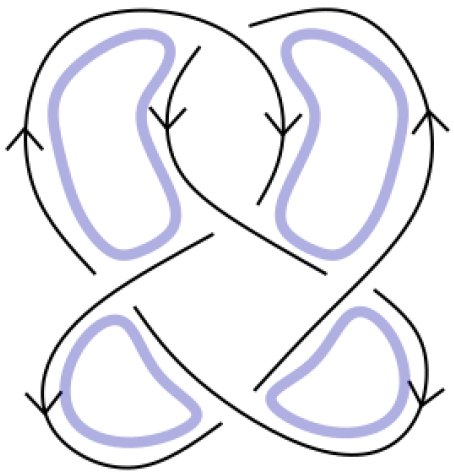
Shuji Yamada

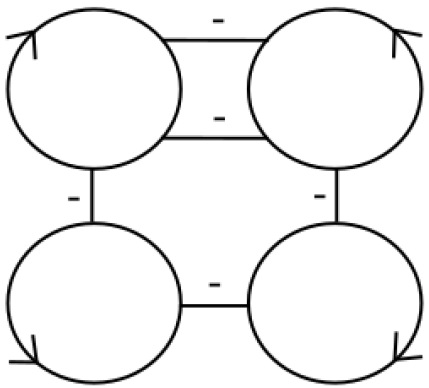
Ana Wright

March 4, 2019

Seifert Diagrams



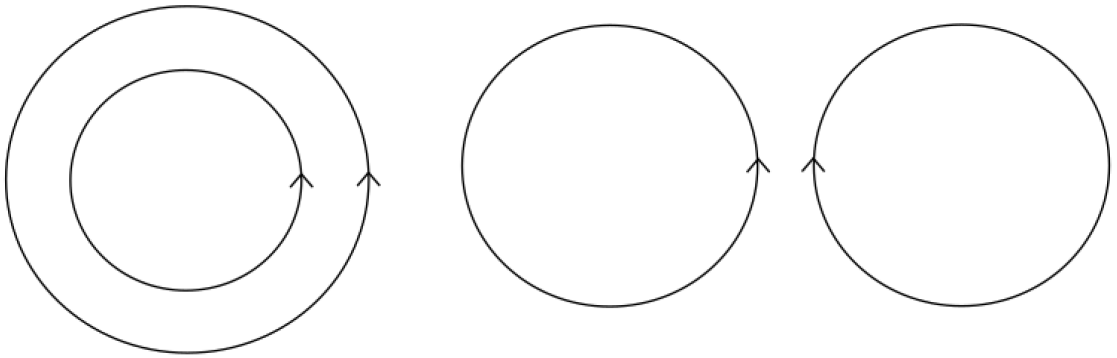




Link Invariant: Minimal number of Seifert circles over all diagrams of a link.

Seifert Diagrams

Coherent Circles



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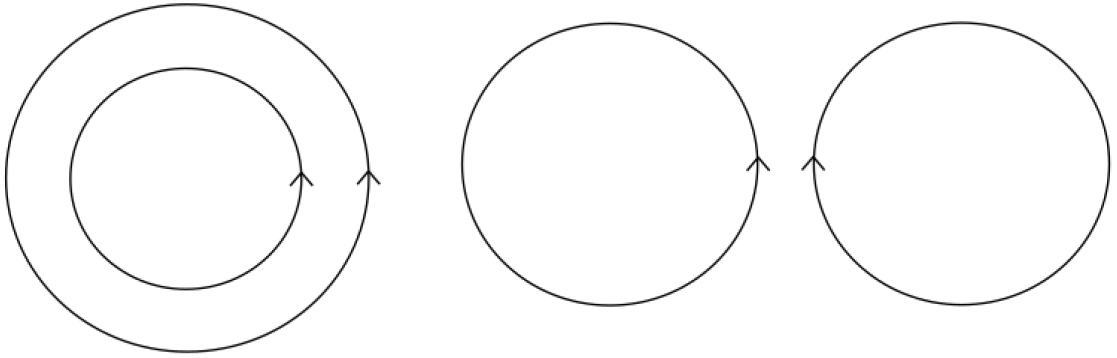
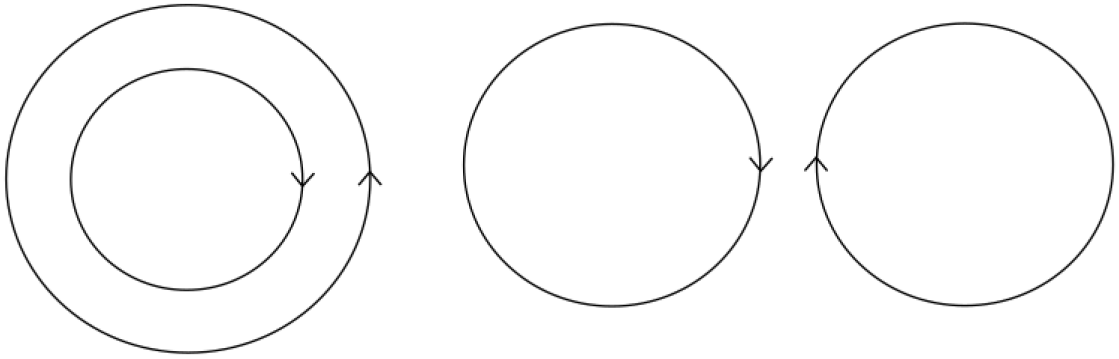
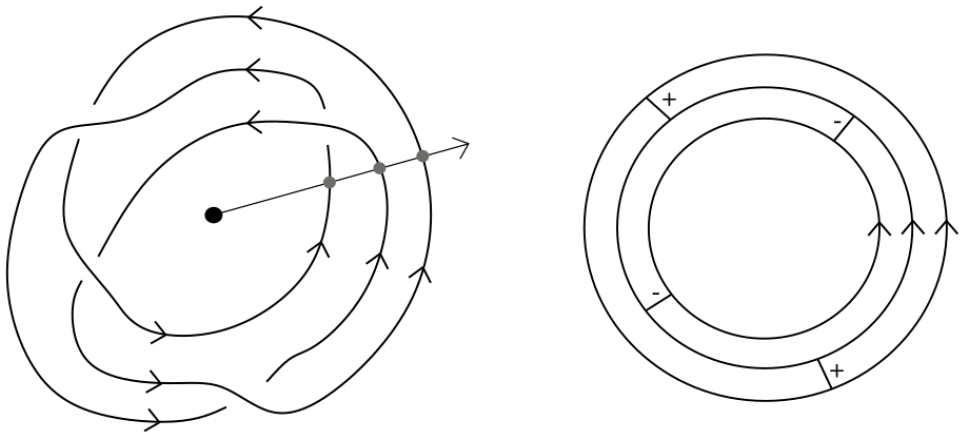


Anti-coherent Circles



Closed Braid Form

A link diagram in closed braid form has some center point such that every ray based at the center intersects the link at the same number of points.



This means that the Seifert diagram of a link in closed braid form is a collection of concentric, coherent Seifert circles.

Link Invariant: The braid index is the minimal number of Seifert circles of the link over all diagrams in closed braid form.

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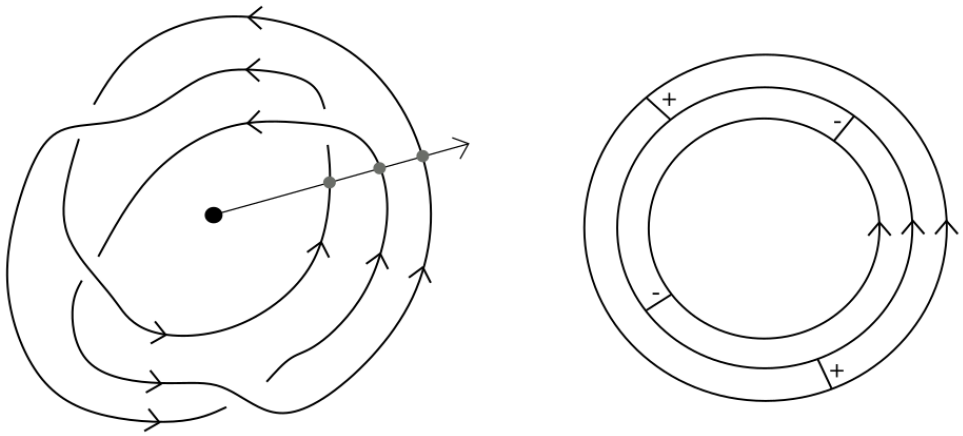

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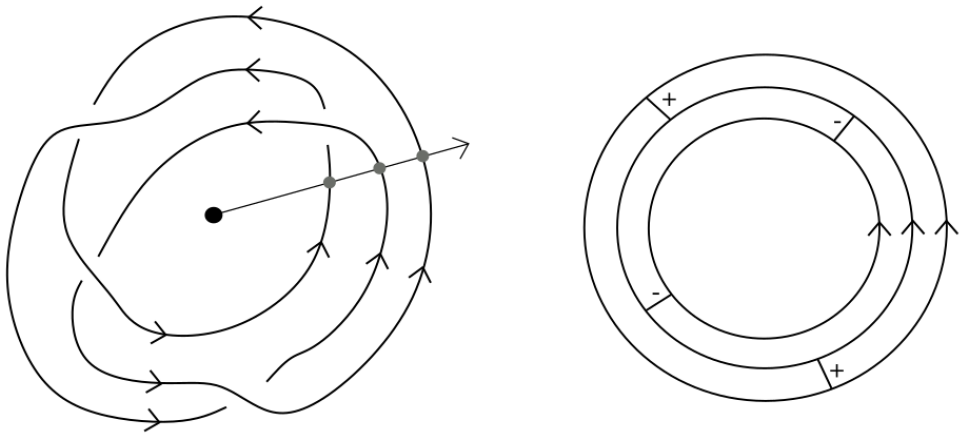

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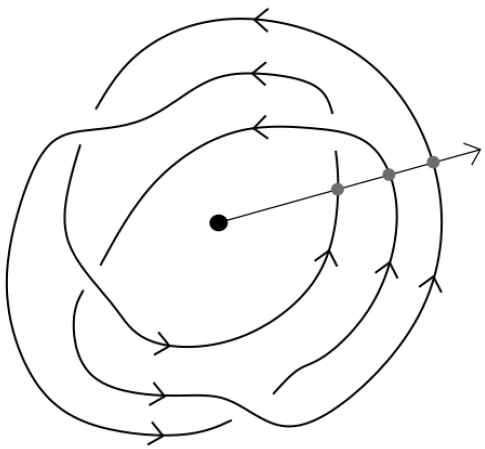
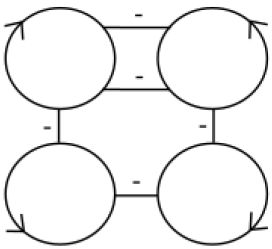
Closed Braid Form

Theorem (Alexander (1923))

Every link diagram can be transformed into a closed braid by an ambient isotopy.



Alexander's Algorithm



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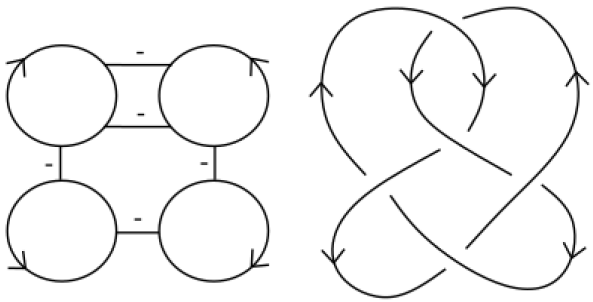

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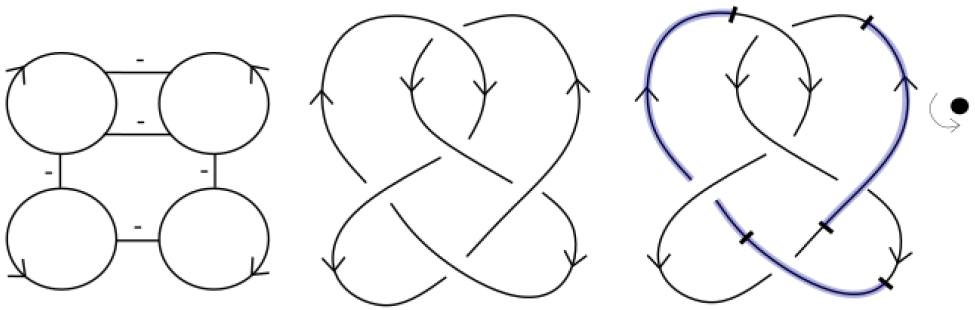

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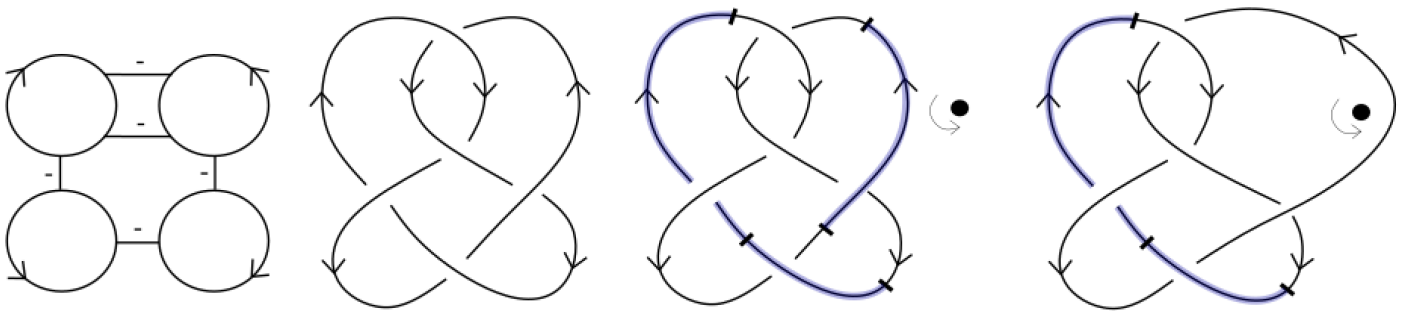

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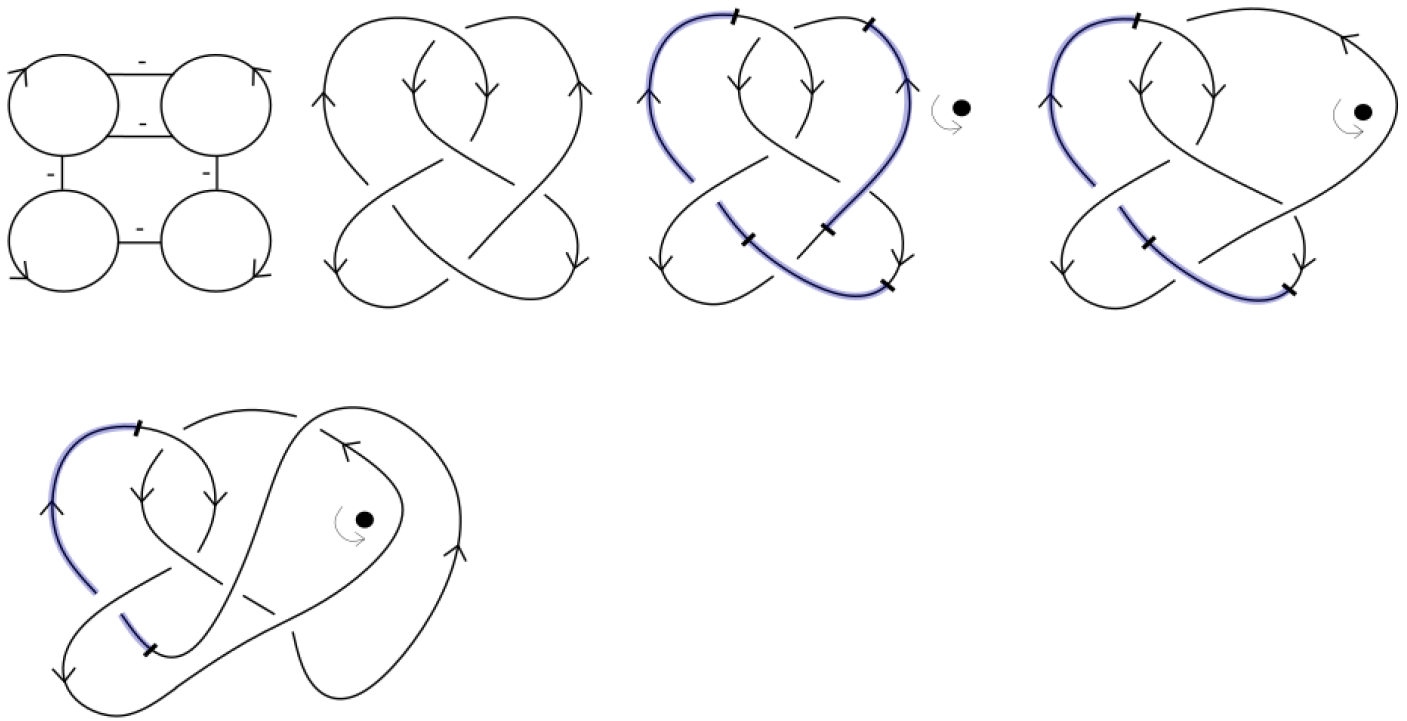

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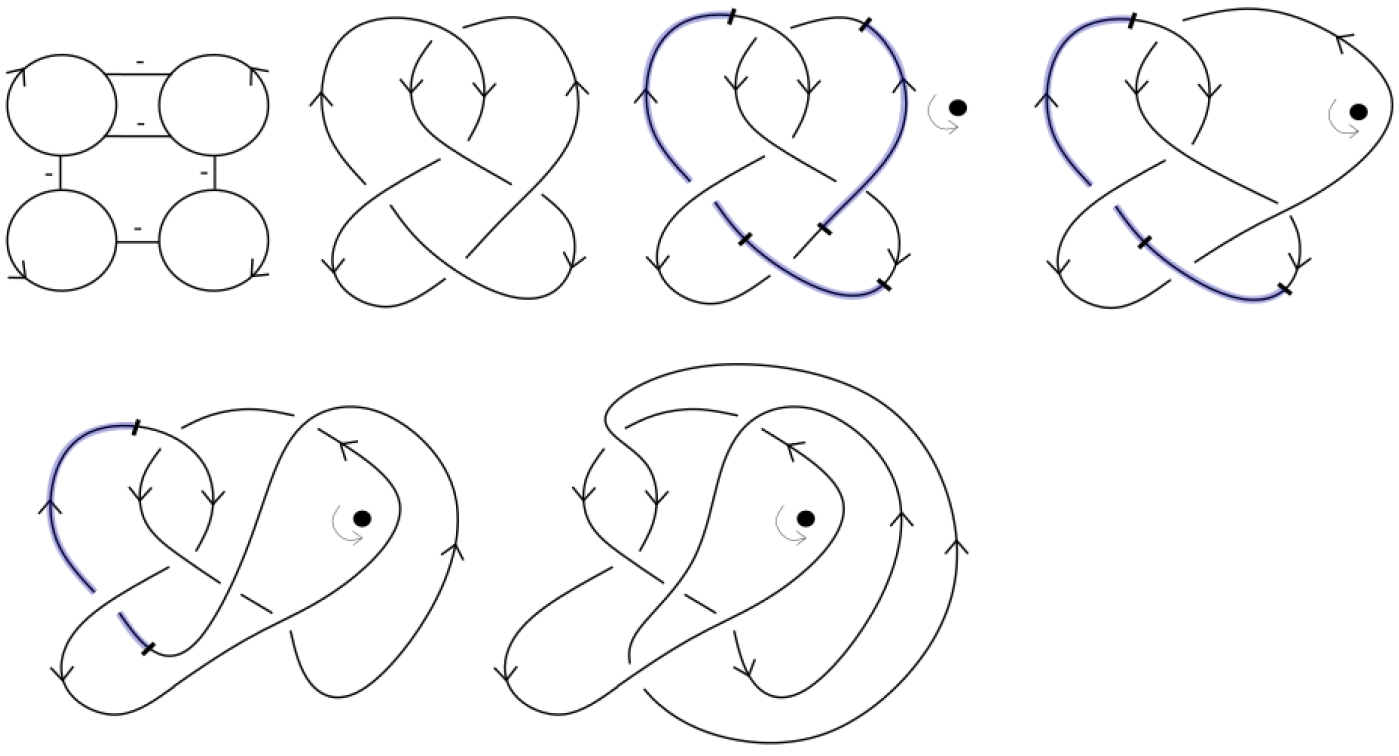

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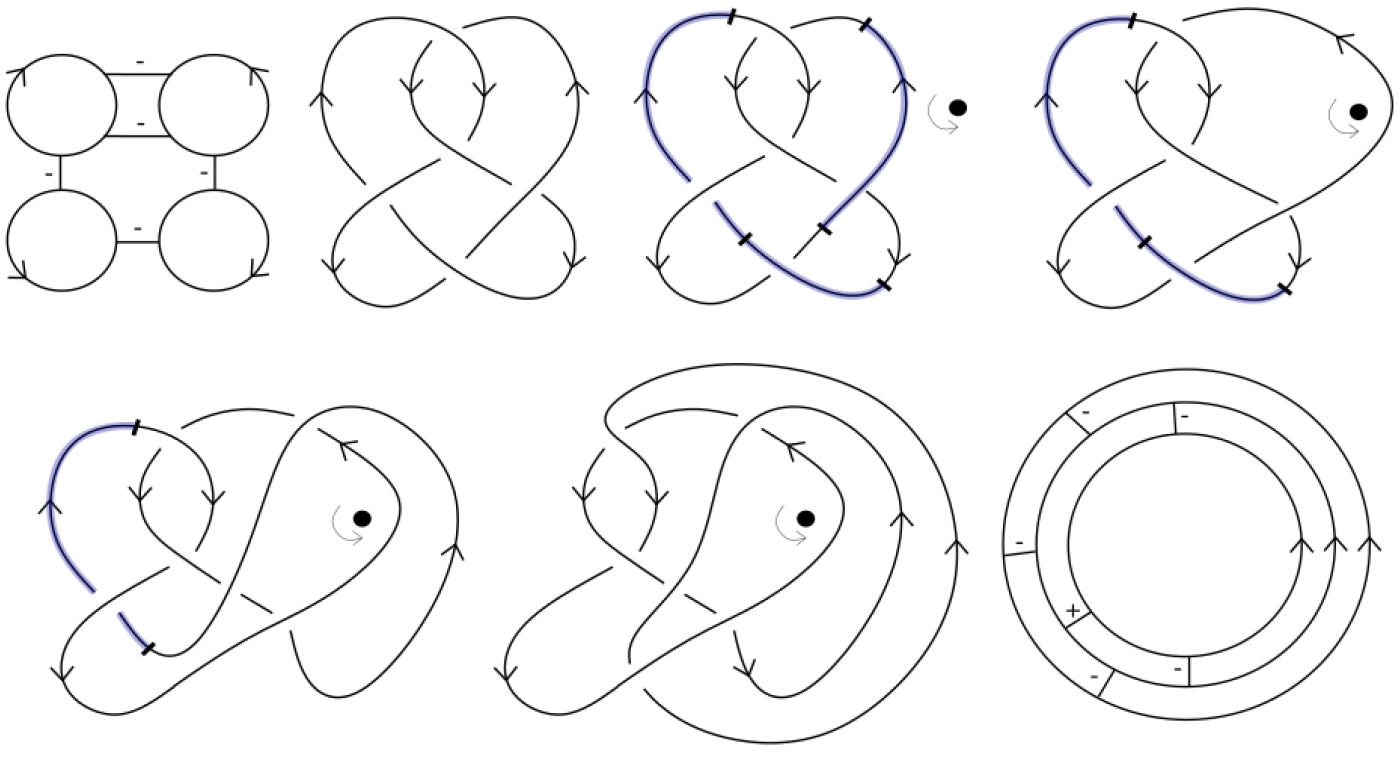
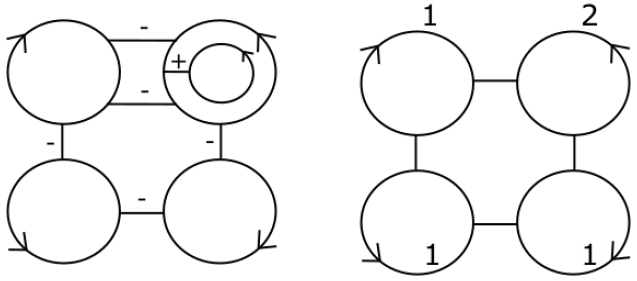
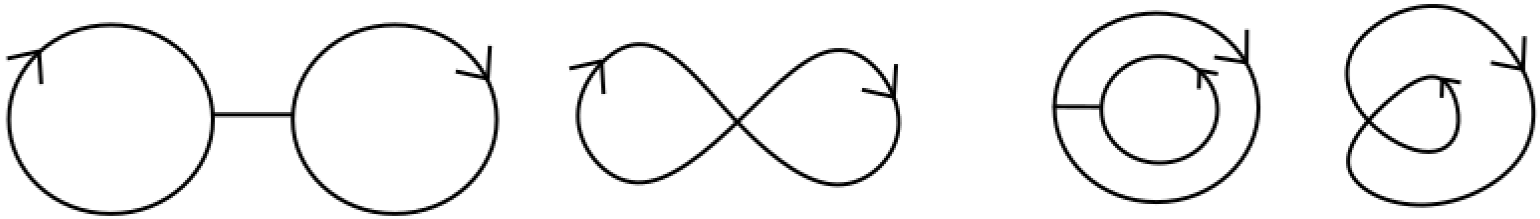


Yamada's Algorithm

Systems of Weighted Seifert Circles



Note: You can't have arcs between anti-coherent Seifert circles.

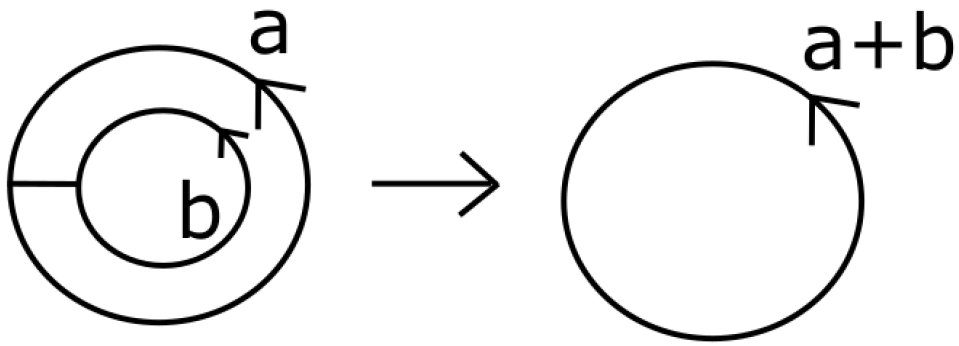


Theorem (Yamada (1987))

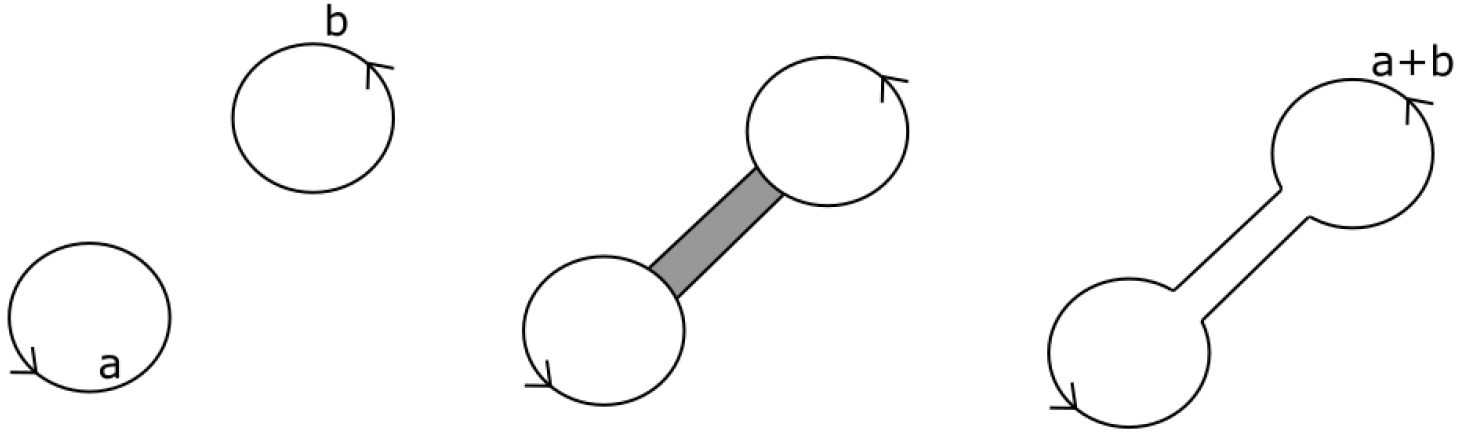
The minimal number of Seifert circles equals the braid index of a link.

Yamada's Algorithm

Bunching Operation Type 1



Bunching Operation Type 2



Yamada's Algorithm

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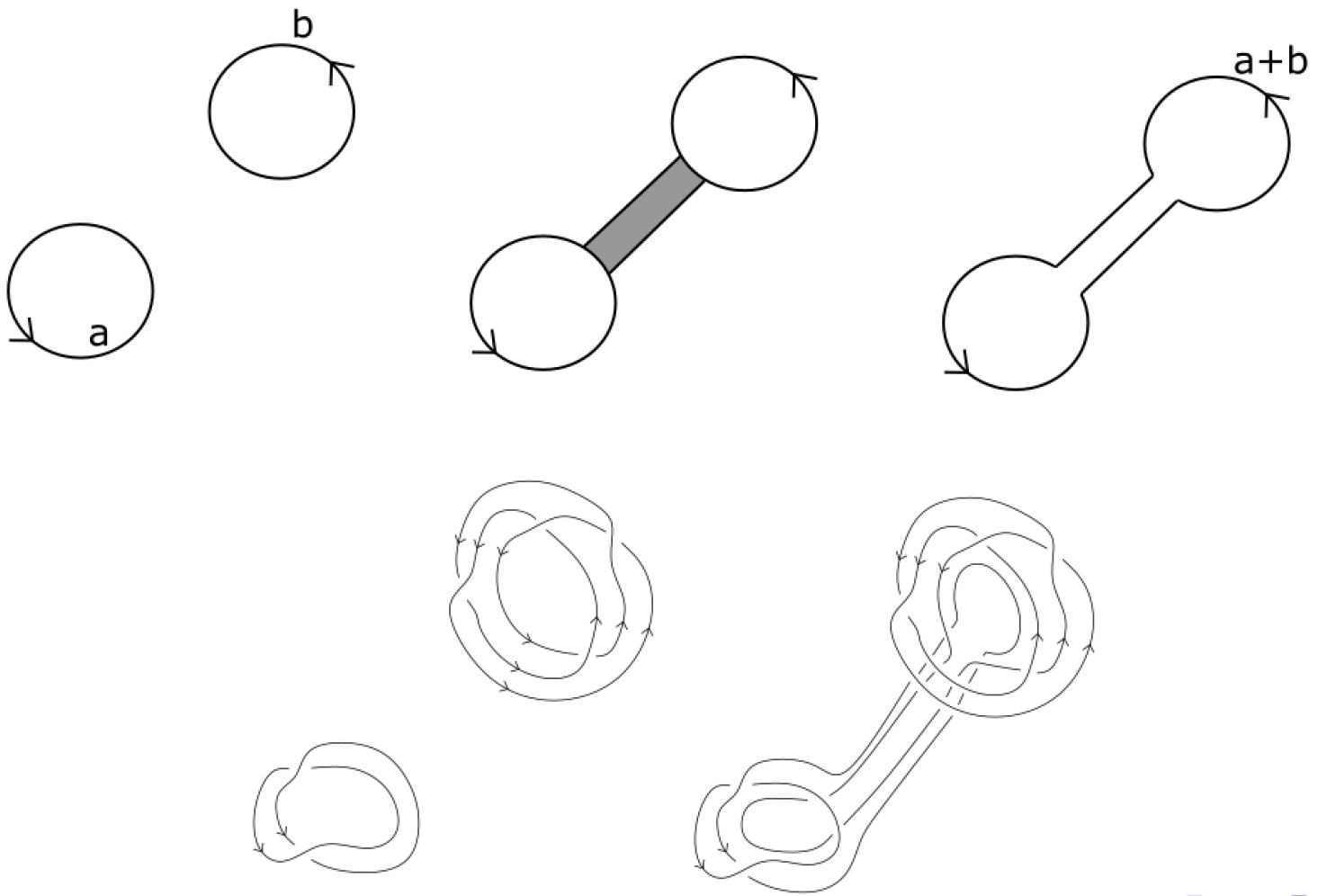

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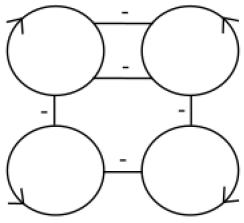

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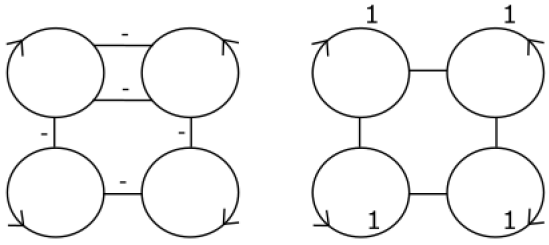

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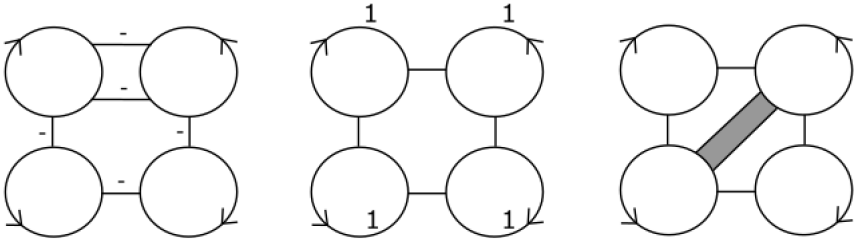

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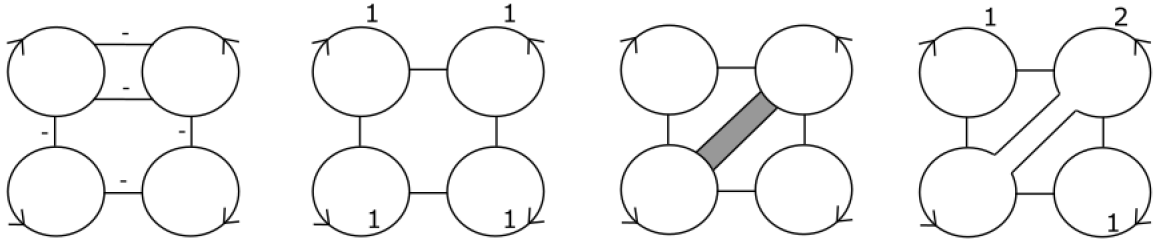

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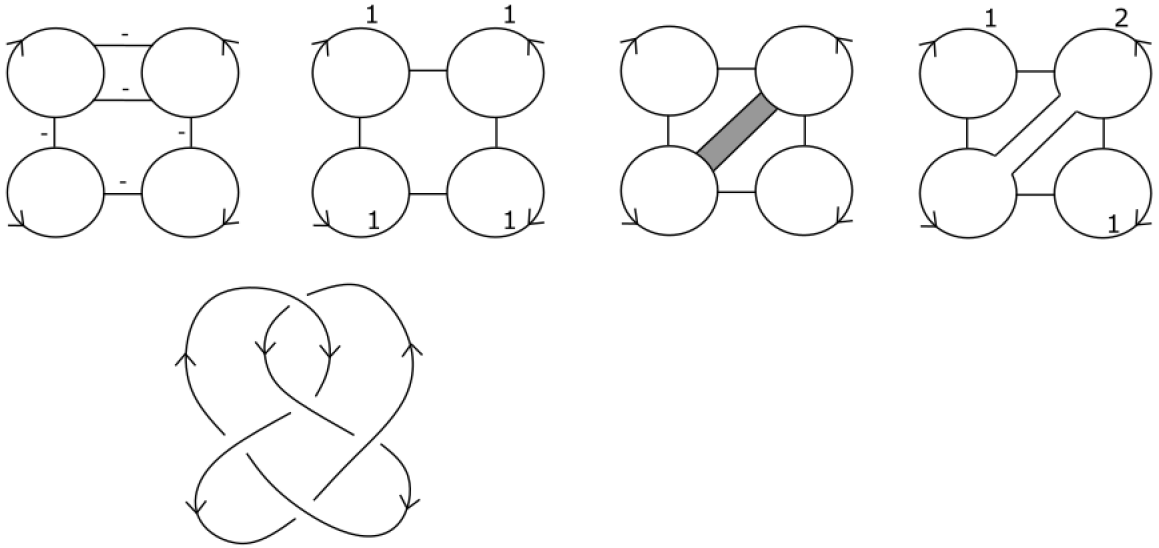

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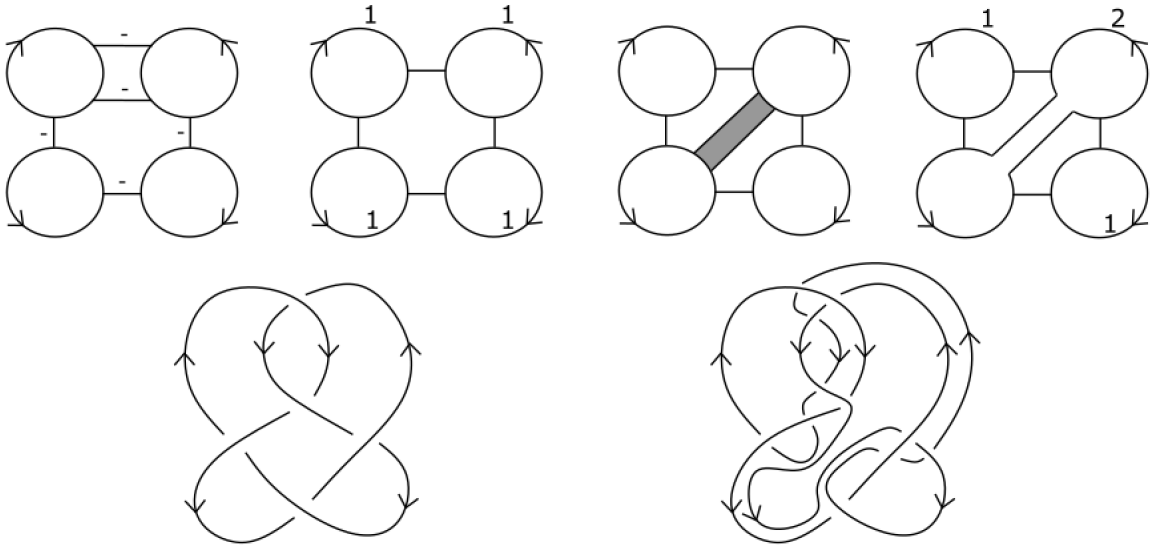

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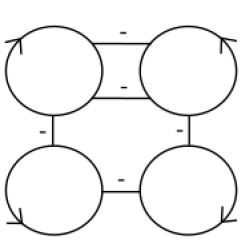

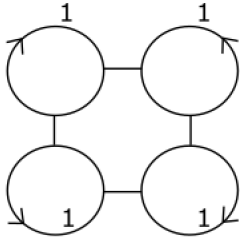

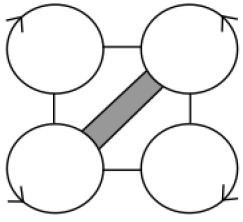

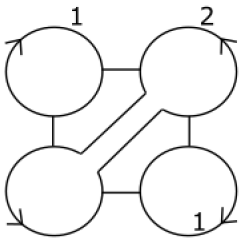

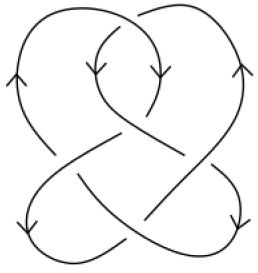

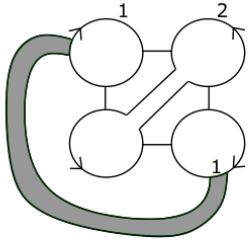

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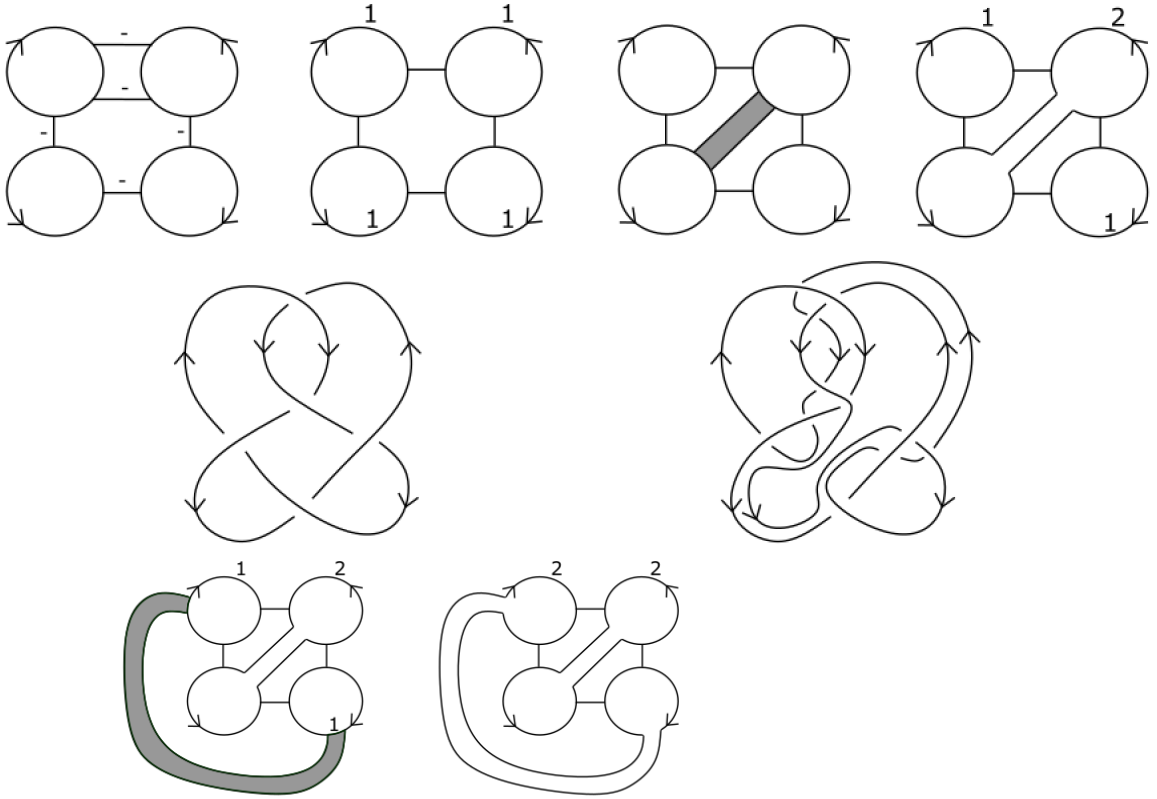

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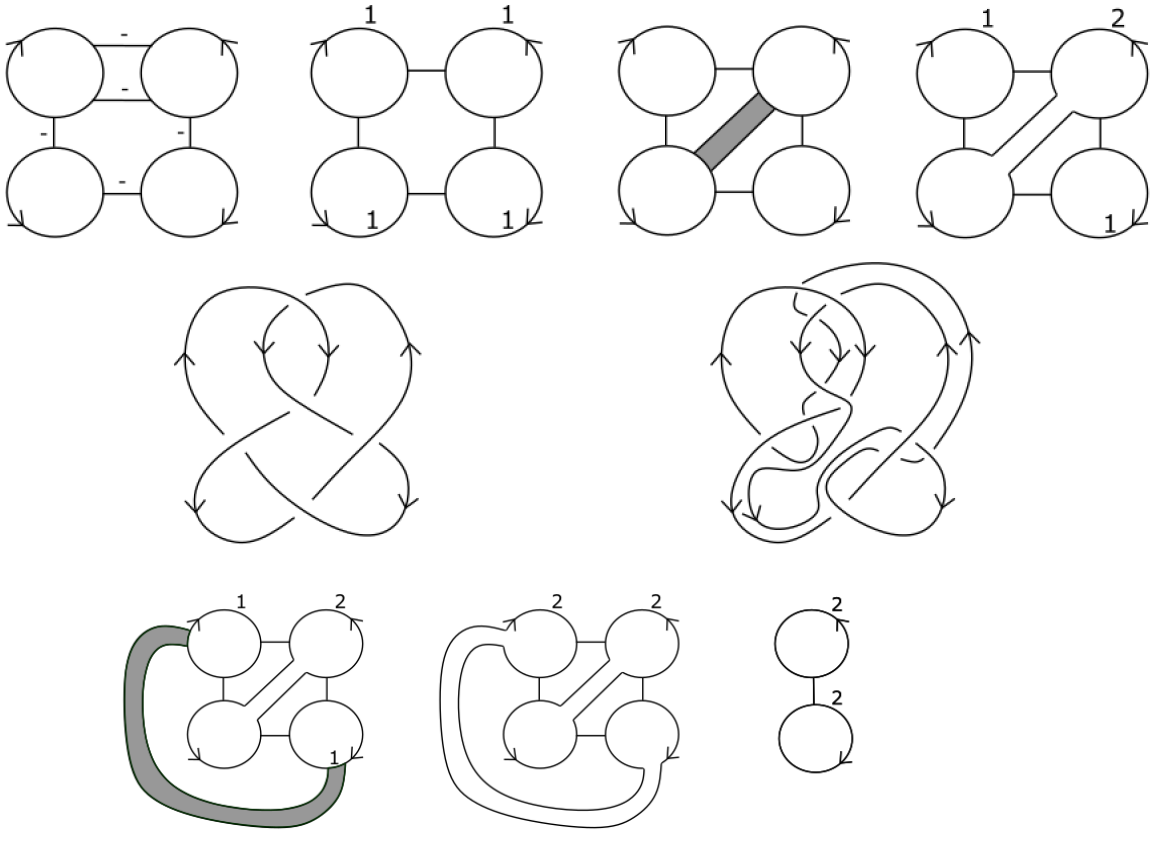

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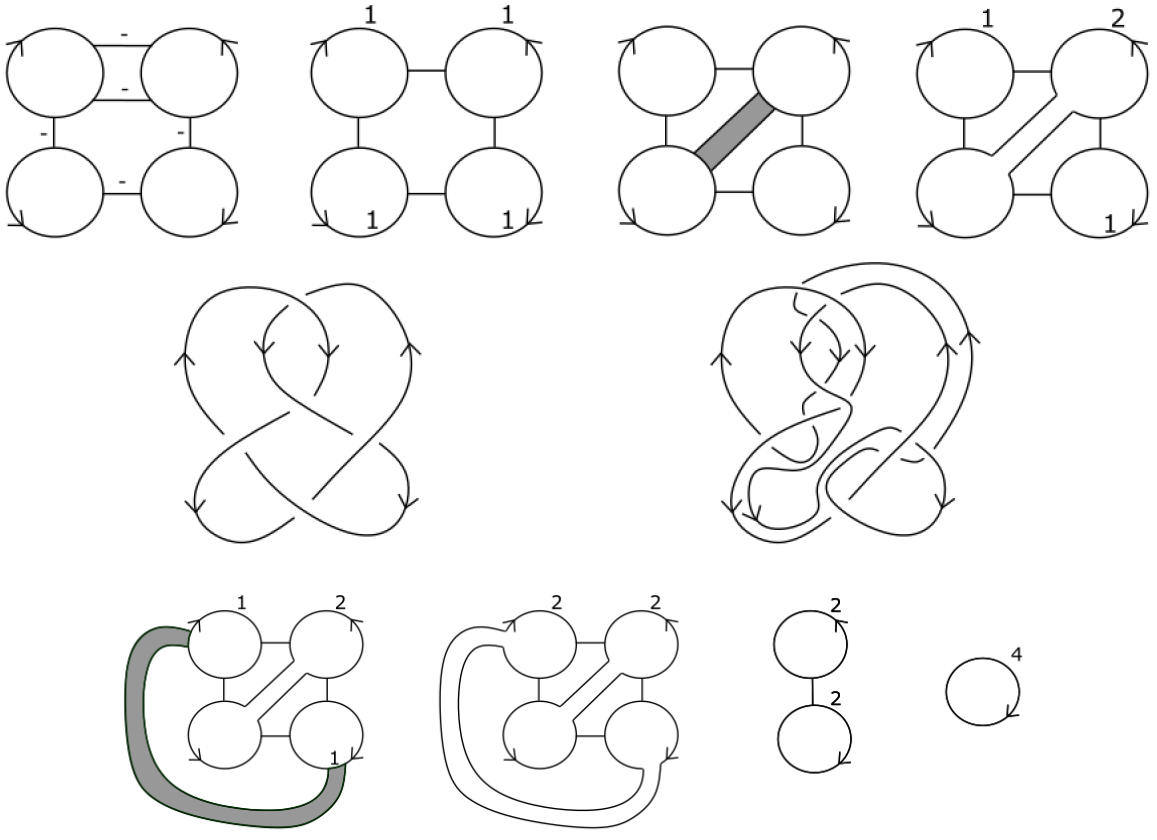

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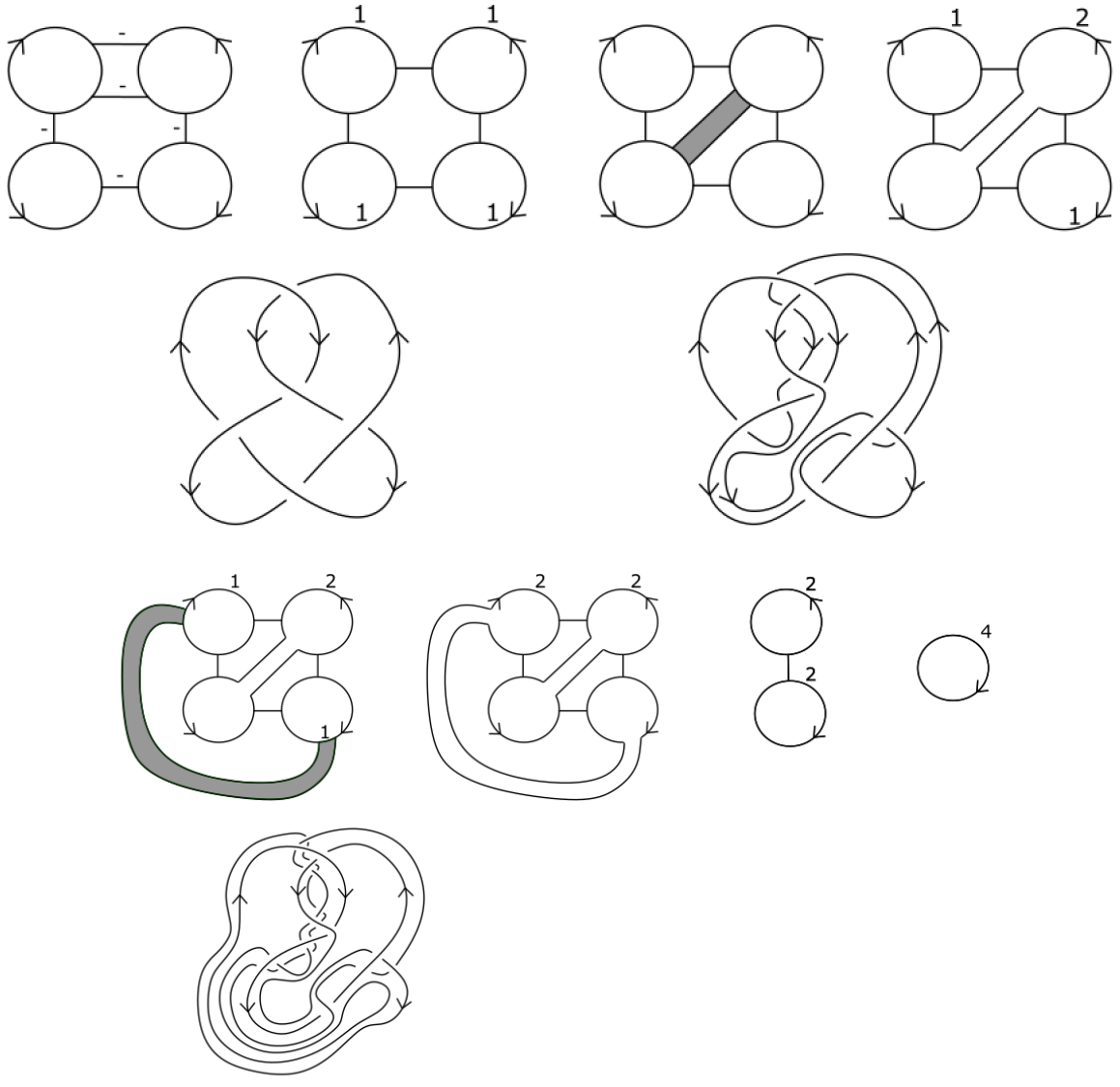

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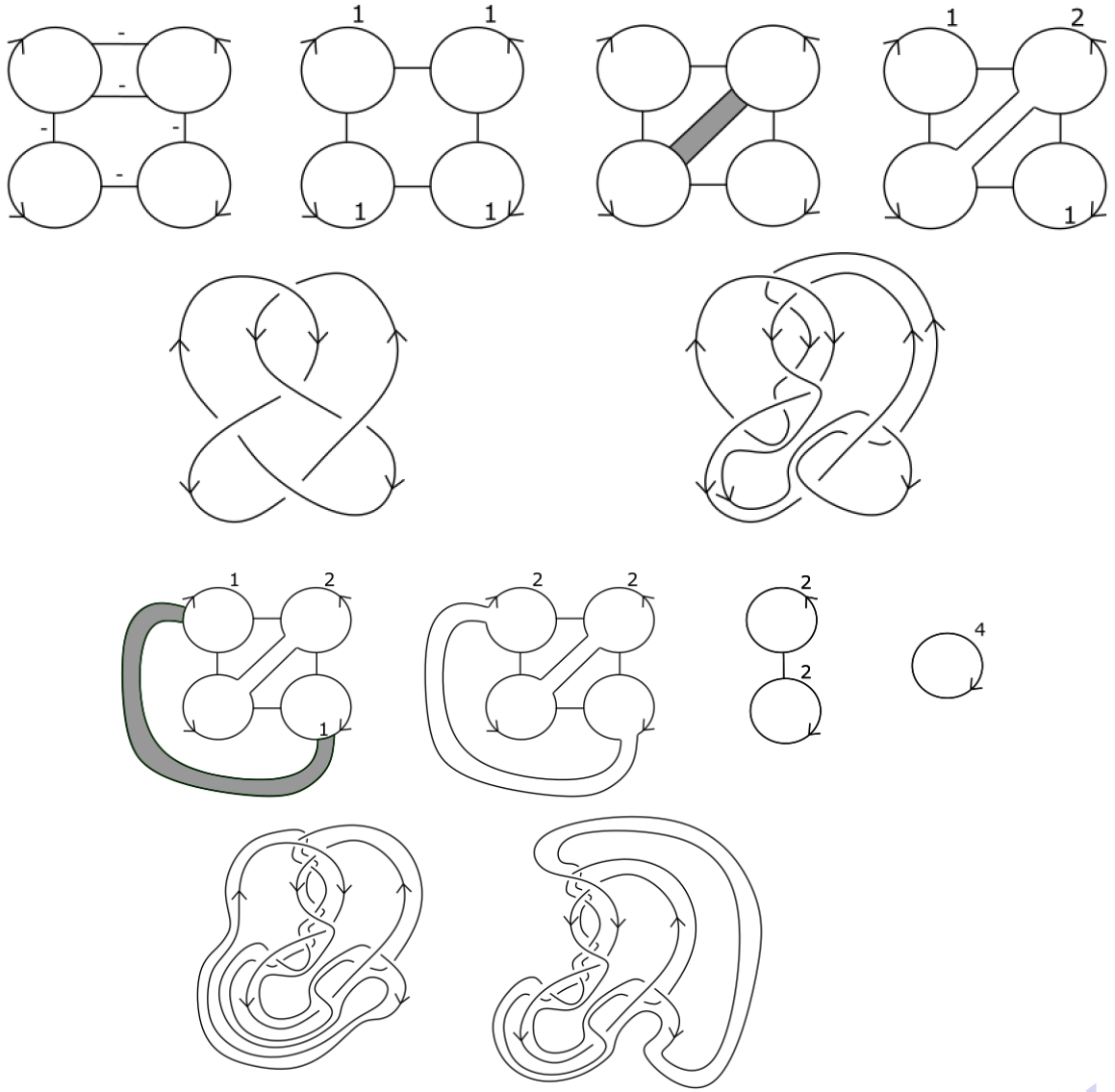
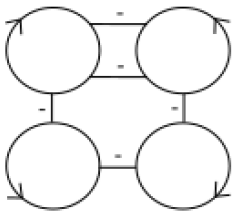


Vogel's Algorithm



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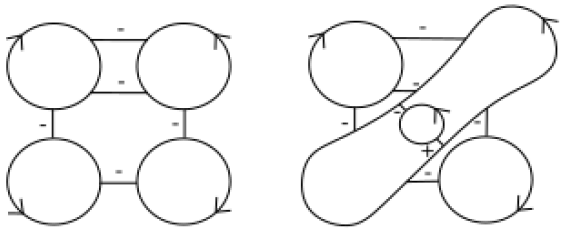

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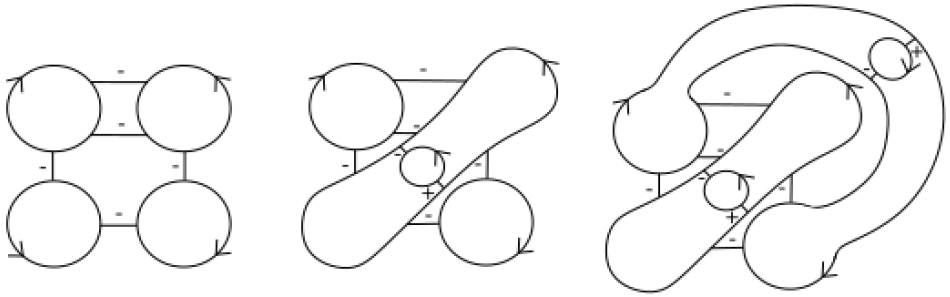

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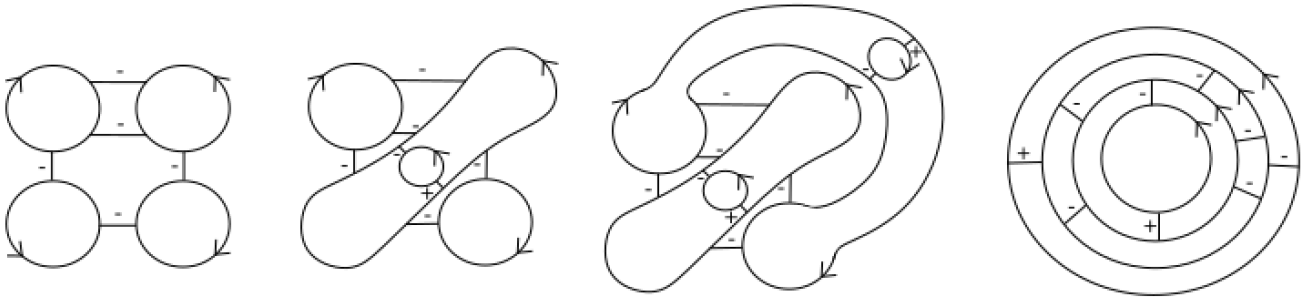

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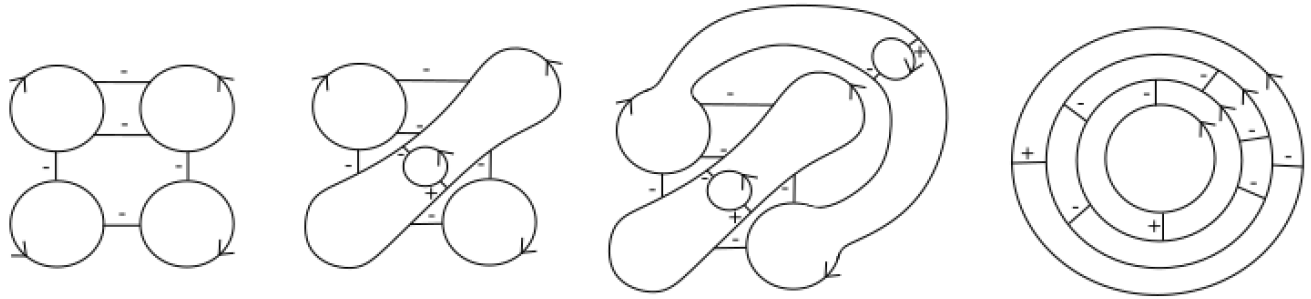

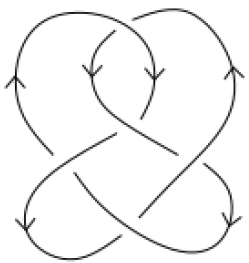

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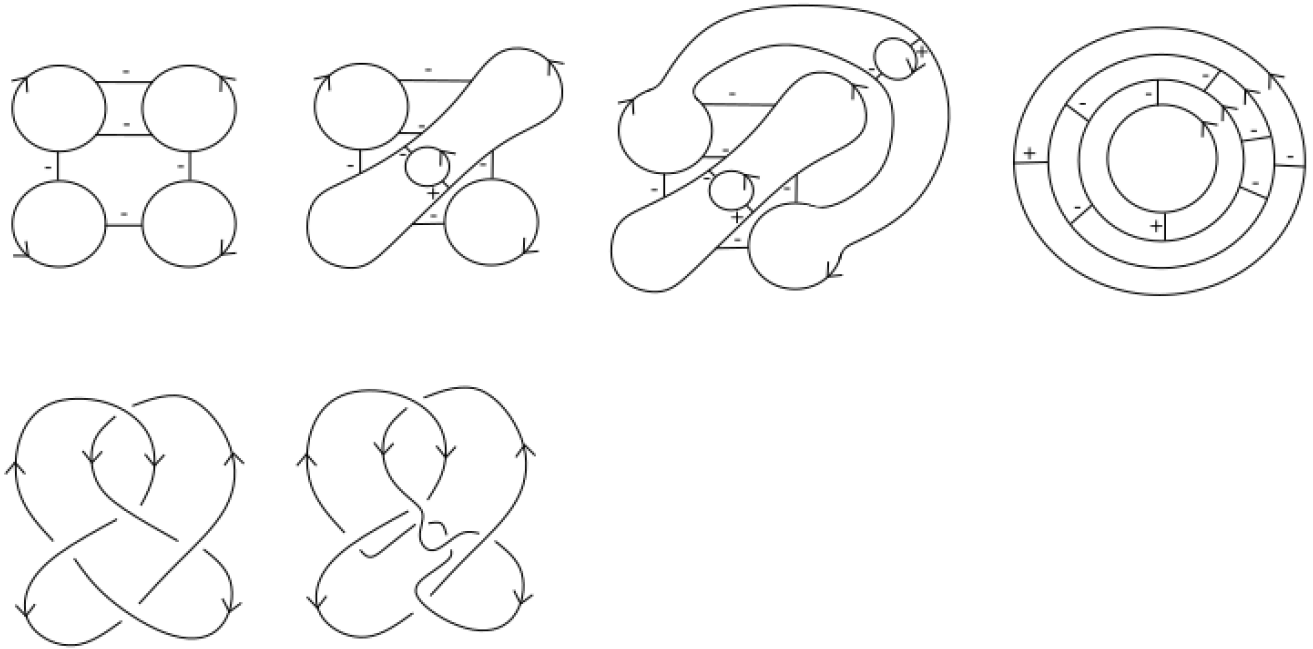

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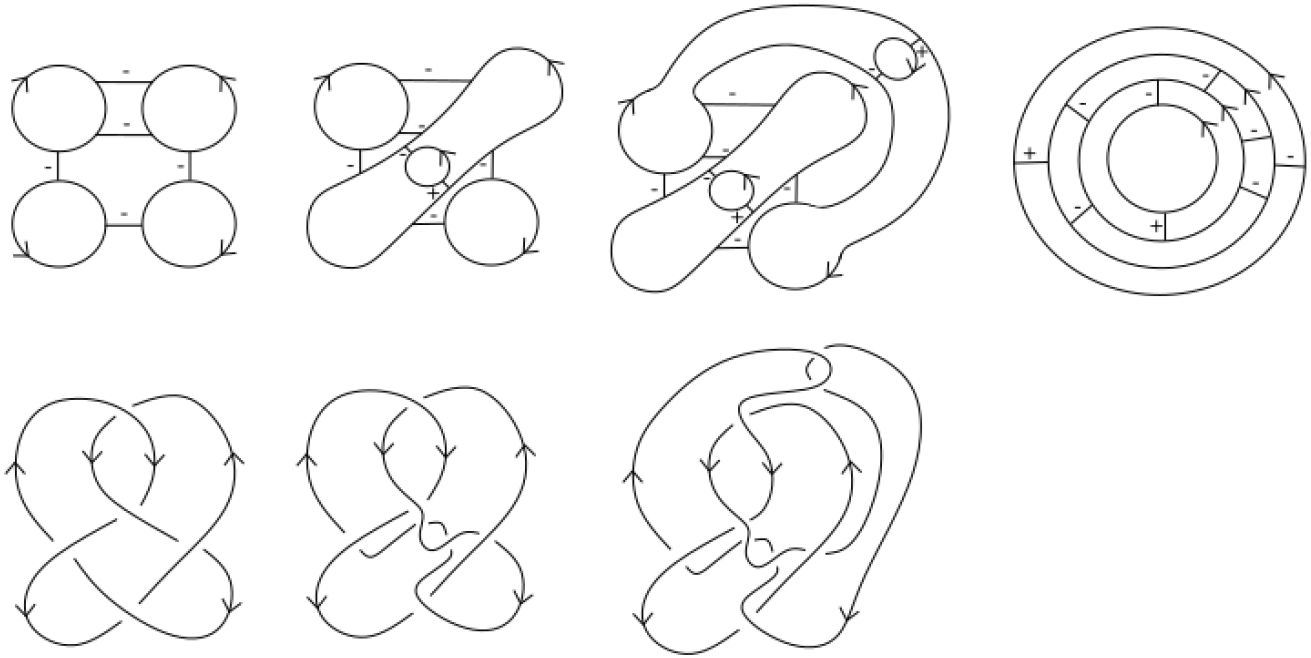

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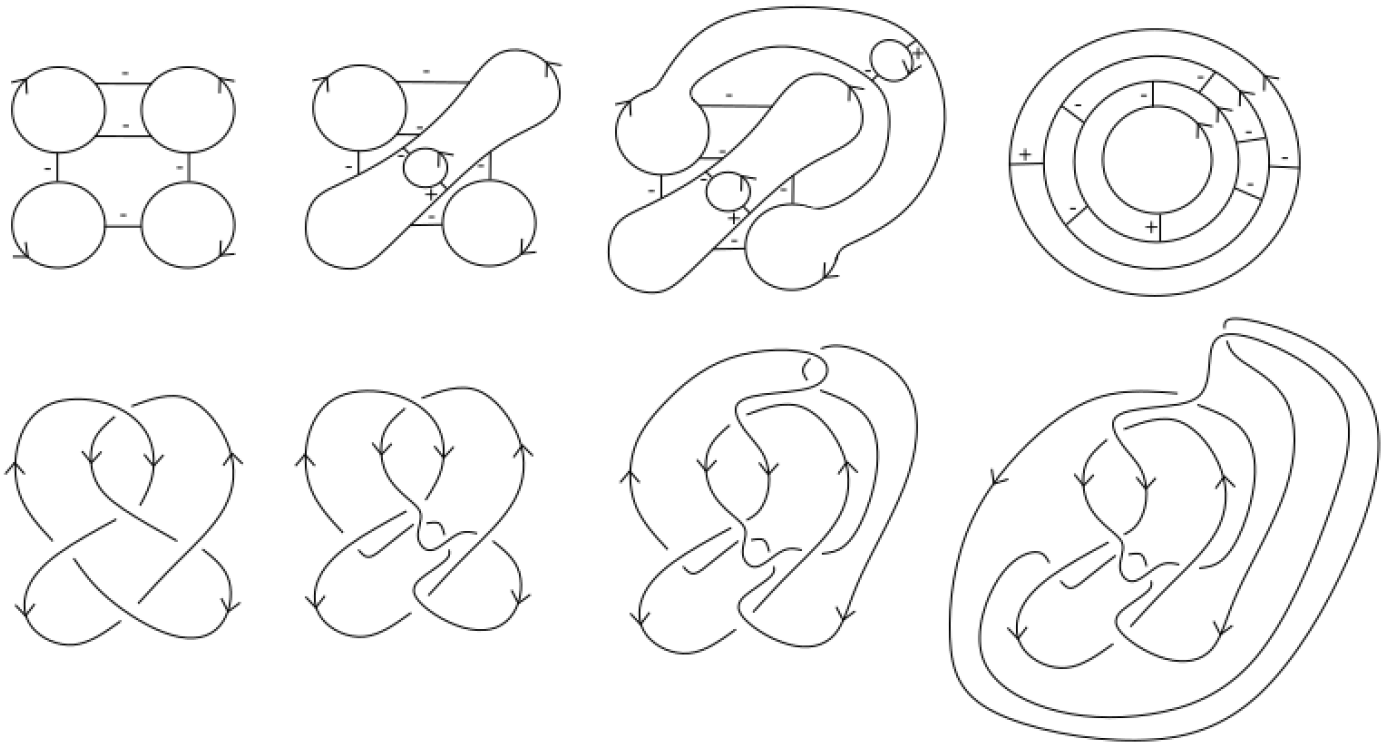
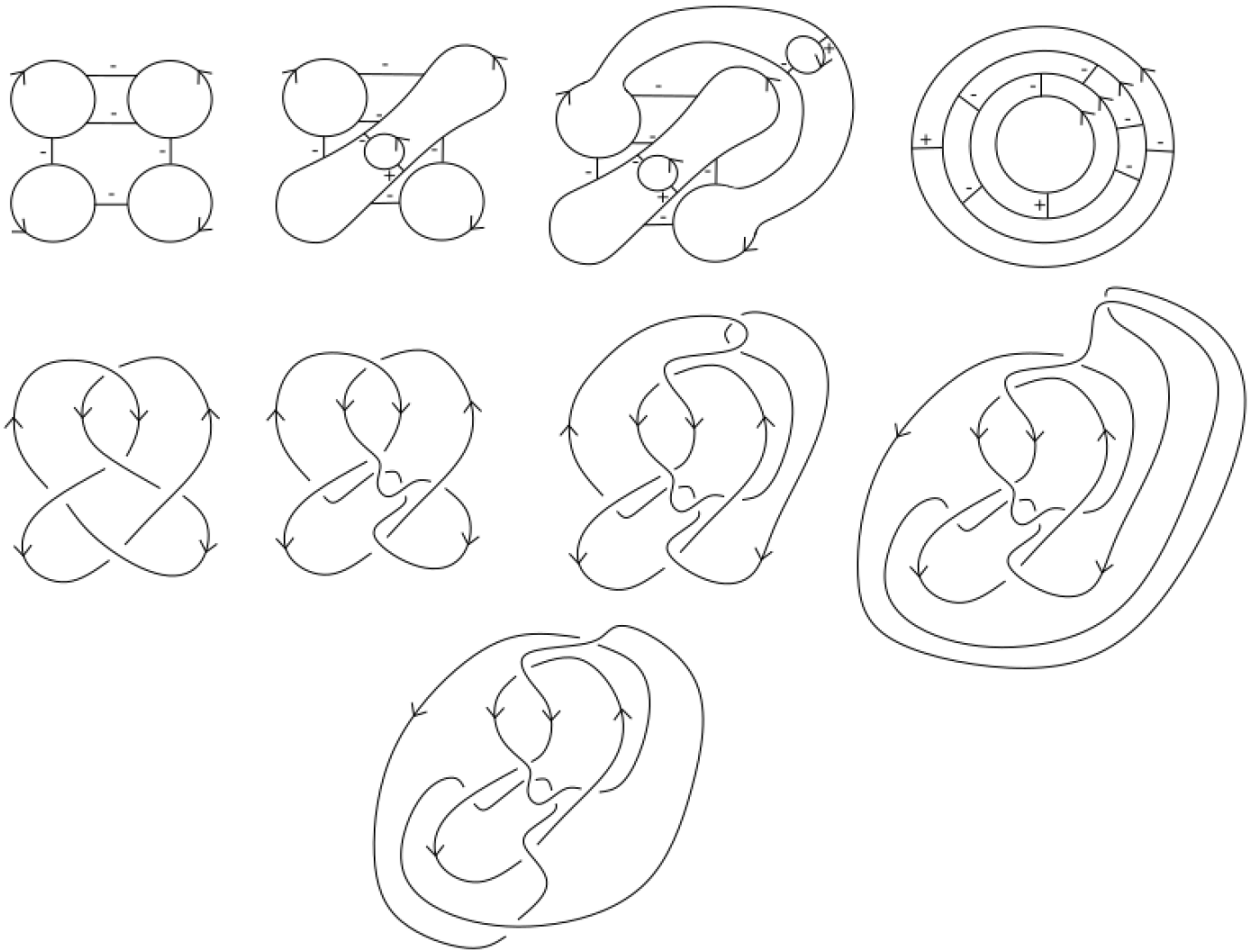
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